N87-20892

THERMAL STABILITY ANALYSIS OF THE FINE STRUCTURE OF SOLAR PROMINENCES.

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SUMMARY

We analyse the linear thermal stability of a 2D periodic structure (alternatively hot and cold) in a uniform magnetic field. The energy equation includes wave heating (assumed proportional to density), radiative cooling and both conduction parallel and orthogonal to magnetic lines. The equilibrium is perturbed at constant gas pressure. With parallel conduction only, it is found to be unstable when the length scale $l_{//}$ is greater than 45 Mm. In that case, orthogonal conduction becomes important and stabilizes the structure when the length scale $l_{\perp}$ is smaller than 5 km. On the other hand, when $l_{\perp}$ is greater than 5 km, the thermal equilibrium is unstable, and the corresponding time scale is about 10^4 s : this result may be compared to observations showing that the lifetime of the fine structure of solar prominences is about one hour ; consequently, our computations suggest that the size of the unresolved threads could be of the order of 10 km only.

FUNDAMENTAL EQUATIONS OF THERMAL EQUILIBRIUM.

We use the 2D "chessboard" of figure 1, which displays a periodic hot (T1) and cold (T2) structure in a uniform magnetic field (this could be the case in the central parts of prominences, see Leroy et al., 1983). We write equations for thermal equilibrium of hot and cold cells, as :

$$\begin{cases} \rho_1 h - \rho_1^2 Q(T_1) - 2\dfrac{F_{//}}{\ell_{//}} - 2\dfrac{F_\perp}{\ell_\perp} = 0 \\ \rho_2 h - \rho_2^2 Q(T_2) + 2\dfrac{F_{//}}{\ell_{//}} + 2\dfrac{F_\perp}{\ell_\perp} = 0 \end{cases}$$

Where $F_{//} = \dfrac{k_{o//}}{3.5}\,\dfrac{T_1^{3.5} - T_2^{3.5}}{\ell_{//}}$ and $F_\perp = -\dfrac{k_{o\perp}}{1.5}\,\dfrac{T_1^{-1.5} - T_2^{-1.5}}{\ell_\perp}$

are respectively the heat flux parallel and orthogonal to the magnetic field. ρ_1 and ρ_2 are the densities of respectively hot and cold cells. $k_{o//}$ and $k_{o\perp}$ are conduction coefficients ; $k_{o\perp}$ depends on the strength of the magnetic field B.

We assume that the gas pressure remains constant ($\rho_1 T_1 = \rho_2 T_2 = \rho T$ = constant and use the cooling function Q (T) given by Hildner (1974). Unknown quantities are h, T_1, T_2. The equilibrium state (h, T_2), when T_1 is fixed, is given by the set of 2 equations above. Possible solutions are shown in figure 2 (top).

ANALYSIS OF THERMAL STABILITY

We perturb the equilibrium at constant gas pressure P and heating h ($T \rightarrow T+\delta T$). Equations are linearized assuming that $\delta T \propto e^{\beta t}$, where β is a growth rate. We assume also that there is no motion in a direction perpendicular to the magnetic field, so that $l_\perp$ = constant. Hence, mass conservation gives $l_{//}\rho$ = constant. We get two solutions for β : the first one is always negative (stable solution), but the second one can either be positive (unstable) or negative (stable). It depends on the values of equilibrium parameters $l_{//}$, $l_\perp$ and T_1. The magnetic field strength B was kept constant (1 Gauss).

RESULTS

Figure 2 (bottom) gives the growth rate as a function fo $l_\perp$ and hot temperature T_1 (parallel conduction was neglected there).
The thermal equilibrium is unstable when $\beta > 0$: this is always the case when $l_\perp$ is too large ($> 10^6$ m). When $l_\perp$ is smaller than 10^5 m, the equilibrium may be stable if T_1 does not exceed a critical value Tmax (Tmax = 10^6 K for $l_\perp = 10^4$ m).

Figure 3 gives β as a function of $l_\perp$ and $l_{//}$ for a 2 D model (T_1 was kept constant). It shows that, when $l_{//} < 45$ Mm, the thermal equilibrium is stable ; when $l_{//} > 45$ Mm, it is unstable, unless $l_\perp < 5$ km. When $l_\perp > 5$ km, the time scale for instability is approximately equal to 10^4 s and corresponds to the observed life time of the fine structure in solar prominences. This result suggests that the size of thin threads could be as small as 10 km.
The temperature of hot cells (T_1) used in the computations was 10^6 K.

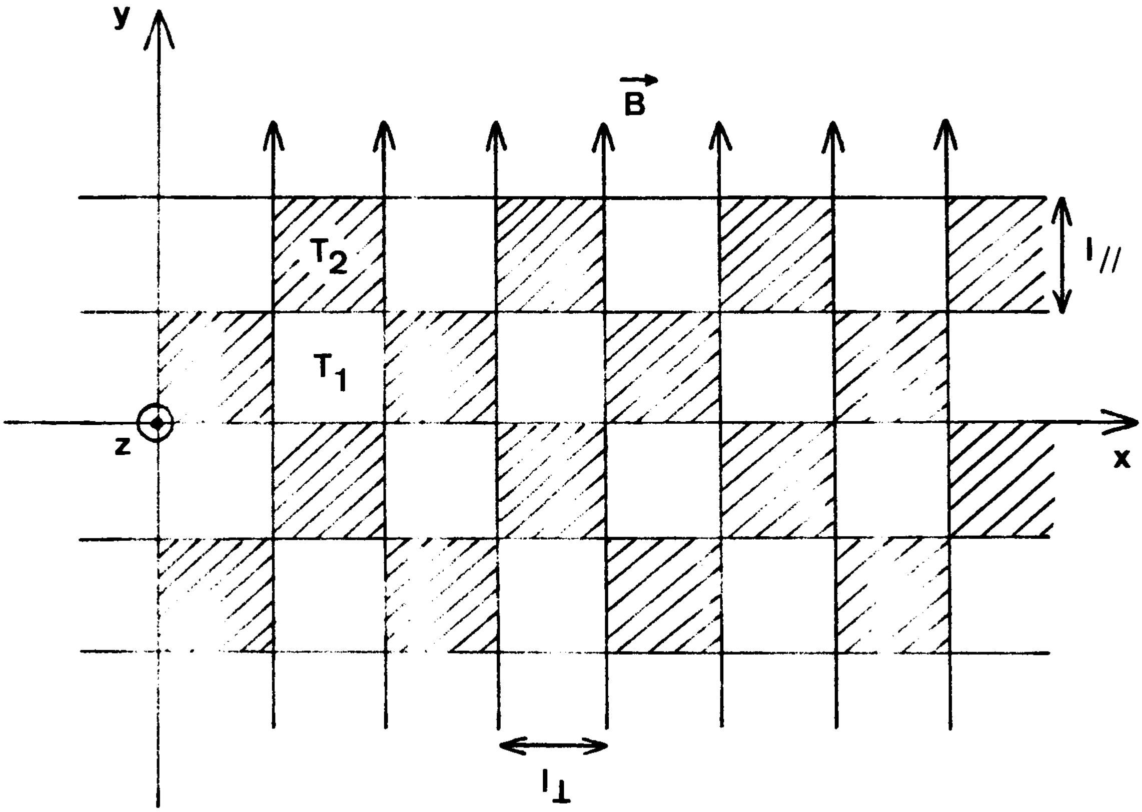
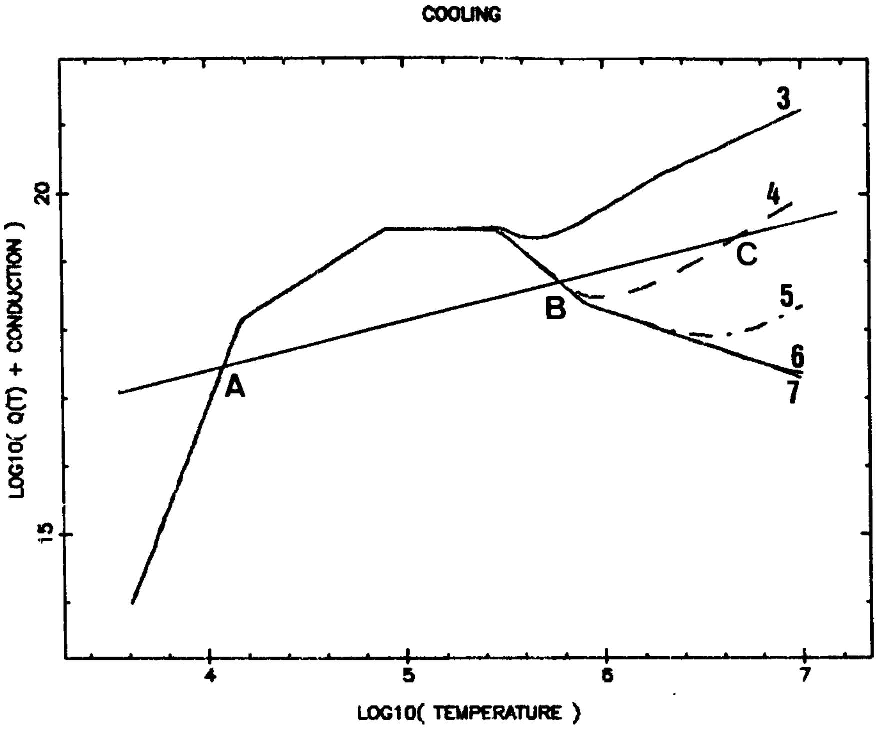


figure 1 : the geometry of the model : the structure is periodic in x direction (length scale $l_\perp$) and y direction (length scale $l_{//}$). The magnetic field B is parallel to y. z is the vertical axis. White areas are hot (T_1) and tenuous (ρ_1) ; dashed cells are cold (T_2 and dense (ρ_2).

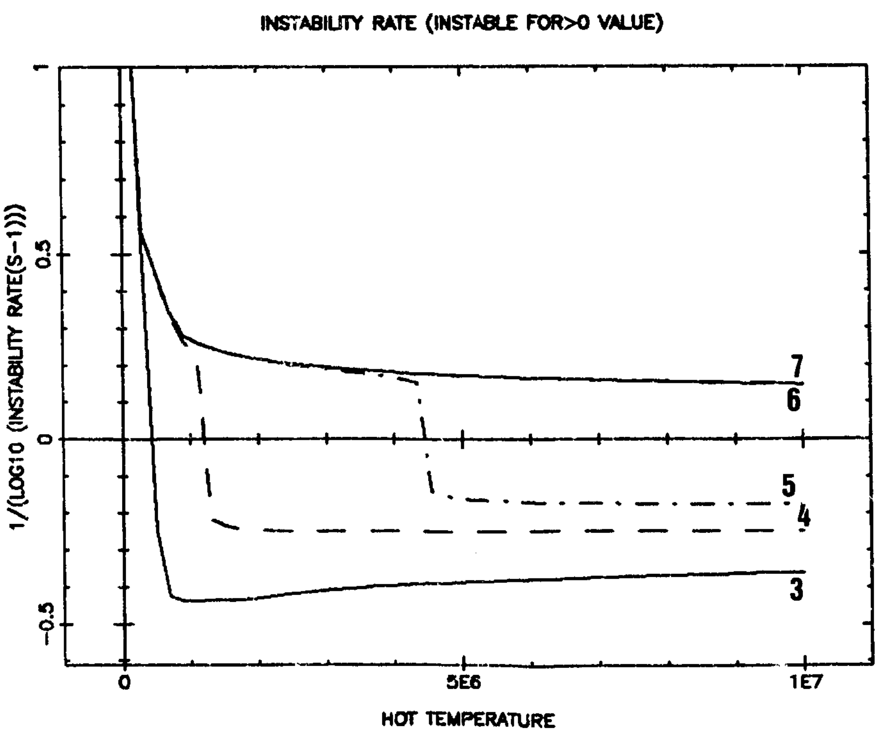
figure 2 : next page

top : log Q (T) + (orthogonal conduction) $/\rho^2$ as a function of log (T) for differnt values of the length scale $l_\perp$ (1D calculation with $l_\perp = 10^3$, 10^4, 10^5, 10^6 and 10^7 m). Equilibrium solutions are located at points A, B, C (intersection with the straight line log (T) + log ($\frac{kh}{pm}$)). When the heating becomes too large, the cold solution A does not exist any more.

bottom : the growth rate as a function of hot temperature T_1 (1 D calculation) The function sgn (β) / log $|\beta|$ is displayed for different values of $l_\perp$ (same as above).



COOLING
LOG10(Q(T) + CONDUCTION)
20
15
4
5
6
7
LOG10(TEMPERATURE)
A
B
C
3
4
5
6
7



INSTABILITY RATE (INSTABLE FOR>0 VALUE)
1/(LOG10 (INSTABILITY RATE(S-1)))
0.5
0
-0.5
0
5E6
1E7
HOT TEMPERATURE
7
6
5
4
3

figure 2

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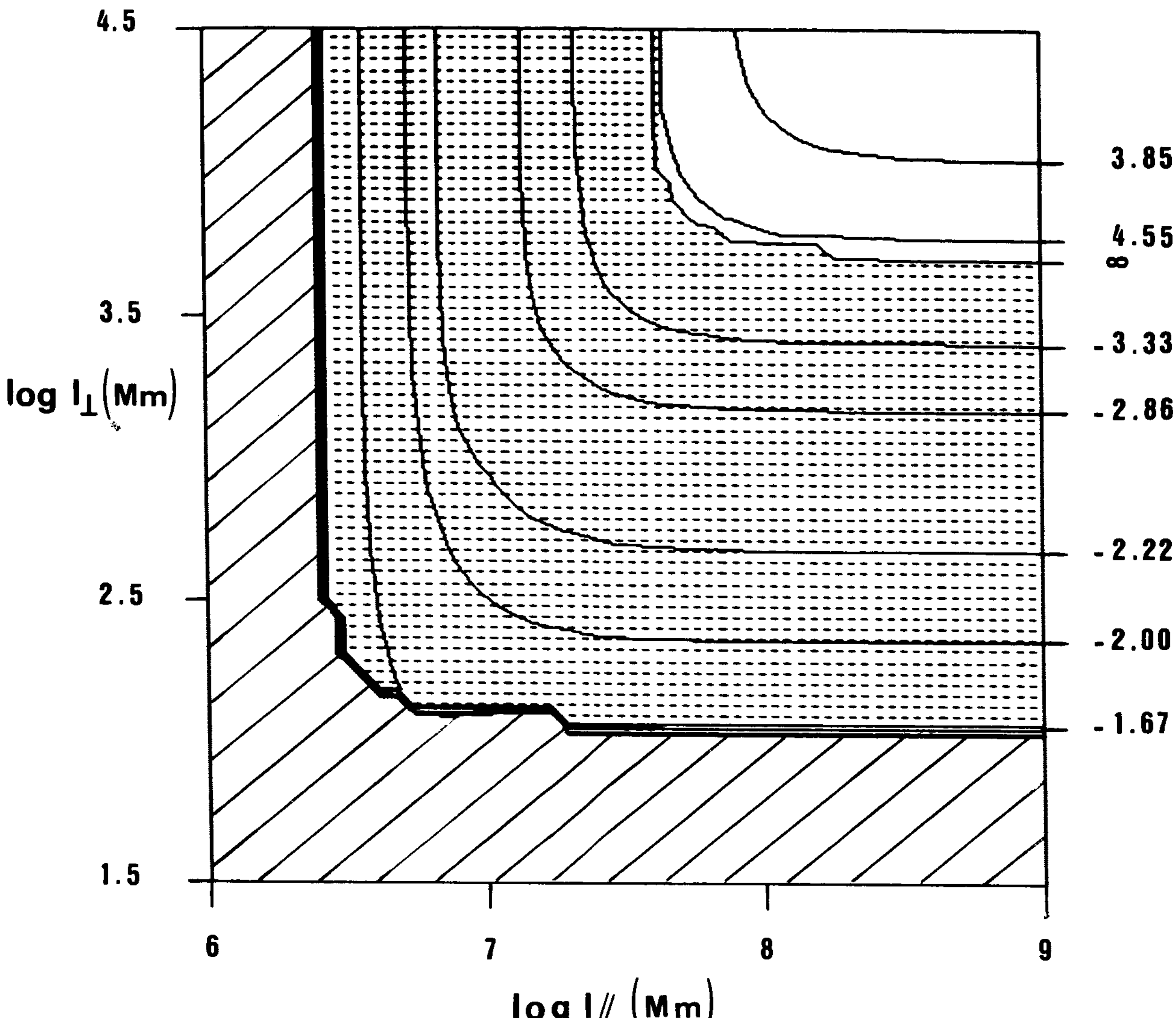

figure 3 : the growth rate for the 2 D calculation as a function of $l_{//}$ and $l_\perp$. Isocontours of sgn (β) log$|\beta|$ are displayed. The dotted area is stable ($\beta < 0$) ; the white one (top) is unstable ($\beta > 0$). The dashed region (left and bottom) represents the domain where a cold equilibrium does not exist (see figure 2).

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